THE CONTACT PROBLEM IN THE THEORY OF CREEP WITH FRICTIONAL FORCES TAKEN INTO ACCOUNT

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A solution is given for the plane contact problem of the theory of creep when frictional forces are taken into account.

We take as our basic physical hypothesis the theory of steady creep as expressed by the equation

$$\varepsilon_i = A\sigma_i^m \tag{0.1}$$

Here ε_i is the intensity of rates of deformation, σ_i is the stress intensity, m is the creep exponent and A is a creep coefficient.

It should be pointed out that the method we develop for solving contact problems in creep theory in no way depends on the choice of the theory of steady creep as our initial physical hypothesis. We could equally have started from the theory of strain-hardening [1] or from the theory of plastic heredity [2].

The contact problem with frictional forces taken into account is solved under conditions of steady creep solely in order to achieve simplicity of presentation.

1. The equilibrium of a half-plane under the simultaneous action of a vertical and a horizontal force applied to its surface, under conditions of steady creep. Consider the problem of the equilibrium of a half-plane loaded simultaneously by a vertical and a horizontal force applied at its surface when creep of the material takes place according to the power law (0.1) for the relation between stresses and rates of deformation.

Take the origin of a cylindrical system of coordinates r, θ and z at the point of application of concentrated forces P and Q on the half-plane, with the directions of r, θ and z as shown in Fig. 1.

According to the theory of steady creep

$$\varepsilon_r = \frac{\varepsilon_i}{\sigma_i}(\sigma_r - \sigma), \qquad \varepsilon_\theta = \frac{\varepsilon_i}{\sigma_i}(\sigma_\theta - \sigma)$$

$$\gamma_{r\theta} = \frac{\varepsilon_i}{\sigma_i}\tau_{r\theta}, \qquad \varepsilon_z = 0, \qquad \sigma = \sigma_z + \frac{1}{2}(\sigma_r + \sigma_\theta) \tag{1.1}$$

These equations assume that the material is incompressible, so that

$$\varepsilon = \varepsilon_r + \varepsilon_\theta = 0 \tag{1.2}$$

Also

$$\sigma_i = \frac{1}{\sqrt{6}}\sqrt{(\sigma_r - \sigma_\theta)^2 + (\sigma_r - \sigma_z)^2 + (\sigma_z - \sigma_\theta)^2 + 6\tau_{r\theta}^2} \tag{1.3}$$

$$\varepsilon_i = \frac{1}{\sqrt{6}}\sqrt{(\varepsilon_r - \varepsilon_\theta)^2 + (\varepsilon_z - \varepsilon_r)^2 + (\varepsilon_z - \varepsilon_\theta)^2 + 6\gamma_{r\theta}^2} \tag{1.4}$$

The equilibrium equations in cylindrical coordinates r, θ and z applicable to the present problem are

$$\frac{\partial}{\partial r}(r\sigma_r) + \frac{\partial \tau_{r\theta}}{\partial \theta} - \sigma_\theta = 0, \qquad \frac{\partial \sigma_\theta}{\partial \theta} + r\frac{\partial \tau_{r\theta}}{\partial r} + 2\tau_{r\theta} = 0 \tag{1.5}$$

The relations between the components of the rate of deformation and the components of the displacement rate vector are

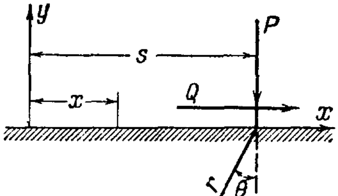

Fig. 1.

$$\varepsilon_r = \frac{\partial u}{\partial r}, \qquad \varepsilon_\theta = \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r} \tag{1.6}$$

$$2\gamma_{r\theta} = \frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r}\frac{\partial u}{\partial \theta}, \qquad \varepsilon_z = \frac{\partial w}{\partial z} = 0$$

where u, v and w are the components of the displacement rate along the coordinate axes r, θ and z, which from now on, for simplicity, will be called simply displacements.

The differential equation of continuity of deformation is then

$$\frac{\partial^2 \varepsilon_r}{\partial \theta^2} + r^2\frac{\partial^2 \varepsilon_\theta}{\partial r^2} + 2r\frac{\partial \varepsilon_\theta}{\partial r} - r\frac{\partial \varepsilon_r}{\partial r} - 2r\frac{\partial^2 \gamma_{r\theta}}{\partial r \partial \theta} - 2\frac{\partial \gamma_{r\theta}}{\partial \theta} = 0 \tag{1.7}$$

The boundary conditions for this problem are

$$\sigma_\theta = \tau_{r\theta} = 0 \quad \text{at} \quad \theta = \pm {}^1\!/_2\,\pi \tag{1.8}$$

i.e. there are no external forces on the free surface of the half-plane, and

$$P = -\int_{-1/2\pi}^{1/2\pi} \sigma_r r \cos\theta\, d\theta, \qquad Q = -\int_{-1/2\pi}^{1/2\pi} \sigma_r r \sin\theta\, d\theta \tag{1.9}$$

for any section of the half-plane bounded by a cylindrical surface r = const.

Suppose that there exists a relation of the form

$$\sigma_i = K_0 \varepsilon_i^{\mu} \qquad (0<\mu<1)$$

$$\left(K_0 = \frac{1}{A^{\mu}},\ \mu = \frac{1}{m}\right) \tag{1.10}$$

between the stress intensities and the deformation rate.

Here K_0 is a creep constant, and μ is the creep exponent determined experimentally by simple creep tests.

TABLE 1.

μ	k	D_3	D_4
1	0	0	0.6366197
	0.1	−0.1	0.6366197
0.70	0	0	0.5772424
	0.1	−0.1289357	0.5798148
0.65	0	0	0.5611840
	0.1	−0.1515839	0.5636824
0.30	0	0.094387	0.3984148
	0.1	0.276217	0.4014432
0.15	0	−0.00741579	0.3241649
	0.1	0.4367790	0.328777
0.20	0	0.2364984	0.3480877
	0.1	0.3558958	0.3520133
0.25	0	0.5701044	0.3727948
	0.1	0.3046570	0.3761803

We shall seek an exact solution to the problem formulated in terms of displacements in the following form:

$$u = \varkappa\, [f_1(r)\, \chi'(\theta) + f_0'(\theta)]$$
$$v = \varkappa\, [f_2(r)\, \chi(\theta) - f_0(\theta)]$$
$$w = 0 \quad (\varkappa \pm 1) \tag{1.11}$$

Here $f_1(r)$, $f_2(r)$, $\chi(\theta)$ and $f_0(\theta)$ are single-valued and continuous functions which can be determined over the whole half-plane

$$-{}^1/_2\pi \leqslant \theta \leqslant {}^1/_2\pi \text{ and } r > 0$$

Setting the shear stress $\tau_{r\theta}(t)$ equal to zero over the whole half-plane, and making use of relations (1.1), (1.2) and (1.6), we obtain the following equations for the determination of the functions $f_0(\theta)$, $\chi(\theta)$, $f_1(r)$ and $f_2(r)$:

$$f_0''(\theta) + f_0(\theta) = 0, \quad r^2 f_1''(r) + r f_1'(r) - (1-\lambda^2)\, f_1(r) = 0$$
$$\chi''(\theta) + \lambda^2 \chi(\theta) = 0, \qquad f_2(r) = -[r f_1'(r) + f_1(r)] \tag{1.12}$$

where λ is a parameter as yet to be found.

Solving equations (1.12) and satisfying the boundary conditions (1.8), the creep law (1.1) and the equilibrium equations (1.5), we obtain

$$\lambda^2 = (2\mu - 1) / \mu^2, \qquad \sigma_\theta = 0 \tag{1.13}$$

We now have the following formulas for finding the displacements of points on the boundary of the half-plane (i.e. at $\theta = \pm 1/2\ \pi$):

$$\begin{aligned} u\,|_{\theta = {}^1/_2\pi} &= B_1 P^m r^{1-m} - C_5, & u\,|_{\theta = +{}^1/_2\pi} &= B_2 P^m r^{1-m} + C_5 \\ v\,|_{\theta = -{}^1/_2\pi} &= A_1 P^m r^{1-m} - C_6, & v\,|_{\theta = {}^1/_2\pi} &= A_2 P^m r^{1-m} + C_6 \quad (0 < \mu < 1) \end{aligned} \tag{1.14}$$

where

$$\begin{aligned} A_1 &= g_1\,(a_2 - a_1), & B_1 &= g_2\,(-b_1 + b_2) \\ A_2 &= -\,g_1\,(a_2 + a_1), & B_2 &= g_2\,(b_1 + b_2) \end{aligned} \quad \left(g_1 = \frac{(m-2)\,D_4{}^m}{(2K_0)^m\,(m-1)\,\lambda}\right)$$

$$\begin{aligned} a_1 &= D_3 \eta_3\,({}^1/_2\,\pi, \mu), & a_2 &= \eta_4\,({}^1/_2\,\pi, \mu) \\ b_1 &= D_3 \eta'\,({}^1/_2\,\pi, \mu), & b_2 &= \eta_4'\,({}^1/_2\,\pi, \mu) \end{aligned} \quad \left(g_2 = -\frac{D_4{}^m}{(2K_0)^m\,(m-1)\,\lambda}\right)$$

$$\eta_3\,(\theta, \mu) = \cos l\theta, \quad \eta_4\,(\theta, \mu) = \sin l\theta, \quad l^2 = (2\mu - 1) / \mu^2 \quad (\mu > {}^1/_2) \tag{1.15}$$

$$\eta_3\,(\theta, \mu) = \cosh \beta\theta, \quad \eta_4\,(\theta, \mu) = \sinh \beta\theta, \quad \beta^2 = (1 - 2\mu) / \mu^2 \quad (\mu < {}^1/_2) \tag{1.16}$$

Also, if we make use of relations (1.13), (1.1), (1.6) and (1.10), we can find an expression for σ_r. Substituting this expression into (1.9), we obtain a system of equations for finding the constants D_3 and D_4

$$\int_{-{}^1/_2\pi}^{{}^1/_2\pi} (-\,D_3 \sin l\theta + \cos l\theta)^\mu\,(k \cos\theta - \sin\theta)\,d\theta = 0 \qquad (\mu > {}^1/_2) \tag{1.17}$$

$$D_4 = \left[\int_{-{}^1/_2\pi}^{{}^1/_2\pi} (-\,D_3 \sin l\theta + \cos l\theta)^\mu \cos\theta\,d\theta\right]^{-1} \qquad (k = Q / P) \tag{1.18}$$

or

$$\int_{-{}^1/_2\pi}^{{}^1/_2\pi} (D_3 \sinh\beta\theta + \cosh\beta\theta)^\mu\,(k \cos\theta - \sin\theta)\,d\theta = 0$$

$$D_4 = \left[\int_{-{}^1/_2\pi}^{{}^1/_2\pi} (D_3 \sinh\beta\theta + \cosh\beta\theta)^\mu \cos\theta\,d\theta\right]^{-1} \quad (\mu < {}^1/_2) \tag{1.19}$$

Table 1 gives values of D_3 and D_4 for various values of the creep exponent μ for $k = 0$ and $k = 0.1$. These values were calculated on the "Aragats" computer in the Computing Center of the Armenian Academy of

Sciences.

2. The plane contact problem in the theory of creep with frictional forces taken into account. 1. *Formulation of the problem and derivation of the basic equation.* We proceed now to the derivation of the basic equation of the contact problem of the theory of creep, taking into account frictional forces.

Suppose that two bodies which possess the property of creep are in contact at a point or along a line and are pressed together by external forces, the resultant of which passes through the origin of coordinates in a direction perpendicular to the x-axis (Fig. 2).

Suppose also that one of these compressed bodies (for example, Body 2) is fixed and that there are no cohesive forces between the bodies, but simply forces of Coulomb friction.

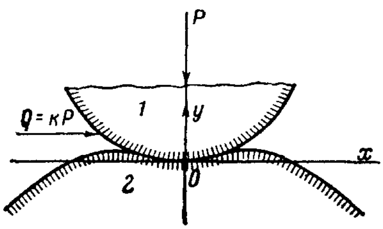

Fig. 2.

In addition, we shall assume, as usual, that Body 1 is in a state of limiting equilibrium.

The relation which must be satisfied by the displacements of points on the area of contact of the bodies is

$$v_1 + v_2 = \delta - f_1^*(x) - f_2^*(x) \quad (2.1)$$

where $\delta = \delta_1 + \delta_2$ is the relative displacement of the bodies in the y-direction and $f_1^*(x)$ and $f_2^*(x)$ are the equations of the surfaces bounding the first and second bodies.

We denote the normal pressure on the area of contact by $p(x)$ and the Coulomb friction force by $q(x) = kp(x)$, where k is the coefficient of friction. Since the area of contact is usually small compared with the dimensions of the bodies themselves, we can assume that the displacements of these bodies will be the same as the displacements of points on the surfaces of two half-planes (an upper and a lower) under the action of the same normal pressure $p(x)$ and Coulomb friction $q(x) = kp(x)$ as the bodies under consideration.

Let us divide up the diagram of pressure $p(x)$ on the area of contact $S(a \leqslant x \leqslant b)$, into elemental strips of width Δs_i and height $p(s_i)$ ($i = 1, \ldots, n$) and consider the action of one of these strips (for example, the ith) on the lower half-plane.

Then vertical and horizontal forces

$$P_i = p(s_i)\,\Delta s_i, \qquad Q_i = kp(s_i)\Delta s_i$$

will be applied at the point $x = s_i$ on the surface of the half-plane.

A point with coordinate x on the surface of this half-plane will be displaced in the direction of the axis Oy by an amount v which, according to (1.14), is given by the formula

$$v = g_1 [a_2 - \operatorname{sign}(s_i - x) a_1] |s_i - x|^{1-m} P_i^m + C \tag{2.2}$$

or in a different form by

$$v^* = h_i p(s_i) \Delta s_i, \qquad v^* = (v - C)^\mu \qquad (m = 1/\mu) \tag{2.3}$$
$$h_i = g_1^\mu [a_2 - \operatorname{sign}(s_i - x) a_1]^\mu |s_i - x|^{\mu - 1}$$

In what follows we shall refer to $v^*(x)$ as the generalized displacement of points on the boundary of the half-plane.

Note that in contrast to the true displacement v, the generalized displacement v^* is linearly dependent on the applied force.

In the case of simultaneous application of a system of forces $P_i = p(s_i)\Delta s_i$ $(i = 1, 2, \ldots, n)$ we are not strictly speaking correct to use the solution already obtained, (2.3), as a Green's function for the present nonlinear problem. Because of the nonlinearity of the problem we have

$$v^* = \sum_{i=1}^{n} h_i p(s_i) \Delta s_i + A_n \tag{2.4}$$

or

$$v = \Big[\sum_{i=1}^{n} h_i p(s_i) \Delta s_i + A_n\Big]^m + C \tag{2.5}$$

where, in general, the quatity A_n for an arbitrary m is nonzero by virtue of the nonlinearity of interaction of the forces. At present an exact solution to this problem presents insuperable mathematical difficulties of principle. In order to derive an approximate solution we make use of the previous linear relation between the generalized displacement v^* and the applied force and proceed as follows.

It follows from the linearity of the problem [3] that in using the relation (3.65) of [3] to find the pressure under a die, $A_n \equiv 0$ when $m = 1$. In the case when $m = 0$ it follows from [3] that the pressure distribution under the die, found on the assumption that $A_n = 0$, coincides with that corresponding to the familiar solution of Prandtl [5].

It is therefore natural to suppose that for an arbitrary m within the range $0 < m < 1$ the approximate solution derived on the assumption that $A_n \equiv 0$ will not differ appreciably from the exact solution. We shall therefore take $A_n = 0$ in (2.4) and (2.5) with an arbitrary value of m within the range $0 < m < 1$, i.e.

$$v^* = g_1{}^{\mu} \sum_{i=1}^{n} [a_2 - \text{sign}\,(s_i - x)\, a_1](s_i - x)^{\mu-1}\, p\,(s_i)\, \Delta\, s_i \qquad (2.6)$$

We should emphasize once more that nowhere within the body does there exist the linear relation between true displacements v and applied force on which depends the validity of the approximate superposition of the generalized displacements v^* on the area of contact $S(a \leqslant x \leqslant b)$.

In (2.6), in the limit as $\Delta s \to 0$, we finally obtain the following expression for the displacements v of points on the area of contact $S(a \leqslant x \leqslant b)$:

$$v = g_1 \left[\int\limits_S \frac{[(a_2 - \text{sign}\,(s-x)\, a_1]^{\mu}\, p\,(s)\, ds}{|\, s - x\,|^{1-\mu}} \right]^m + C \qquad \left(m = \frac{1}{\mu}\right) \qquad (2.7)$$

Here the constants a_1, a_2 and g_1 can be determined from (1.14).

Making use of (2.1) and (2.7), we obtain the following singular Fredholm integral equation of the first kind for the pressure:

$$\int\limits_S \frac{[a_2 - \text{sign}\,(s-x)\, a_1]^{\mu}\, p\,(s)\, ds}{|\, s - x\,|^{1-\mu}} = F\,(x,\, \gamma) \qquad (\gamma = \text{const}) \qquad (2.8)$$

Here

$$F\,(x,\, \gamma) = [\gamma - f_0\,(x)]^{\mu}, \qquad f_0\,(x) = \frac{f_1^*\,(x) + f_2^*\,(x)}{2g_1} \qquad (2.9)$$

The constant γ in (2.8) will be found later.

Thus the singular integral equation (2.8) is the basic equation of the plane contact problem of the theory of nonlinear creep when frictional forces are taken into account and when a relation of the form (1.10) exists between the intensities of rate of diformation and stress.

2. *Solution of the basic integral equation of the plane contact problem of the theory of creep with frictional forces taken into account.* Suppose that after the bodies are compressed the area of contact S is defined by the interval $-a \leqslant x \leqslant a$ of the x-axis. Then the basic integral equation (2.8) assumes the form

$$\int\limits_{-a}^{a} \frac{[a_2 - \text{sign}\,(s-x)\, a_1]^{\mu}\, p(s) ds}{|\, s - x\,|^{1-\mu}} = F\,(x,\, \gamma) \qquad (2.10)$$

Here $F(x,\, \gamma)$ is a continuous function, $2a$ is the width of the area of contact and γ is a constant which for a given width of contact can be found from the equation of equilibrium

$$P = \int_{-a}^{a} p(x)\,dx \tag{2.11}$$

where P is the resultant of the external forces acting on the compressed body. According to [4] the general solution of equation (2.10) is

$$p(x) = \frac{1}{M'(a)}\left[\frac{d}{dx}\int_{-a}^{x} g(s,x)F(s,\gamma)\,ds\right]_{x=a} g^*(x,a) - \tag{2.12}$$

$$-\int_{x}^{a} g^*(x,u)\frac{d}{du}\left[\frac{1}{M'(u)}\frac{d}{du}\int_{0}^{u} g(s,u)F(s,\gamma)\,ds\right]du$$

Here $g(s, a)$ is the solution to the equation

$$\int_{-a}^{a}\frac{[a_2 - \operatorname{sign}(s-x)\,a_1]^{\mu}\,g(s,a)\,ds}{|s-x|^{1-\mu}} = 1 \tag{2.13}$$

$g^*(s, a)$ is the solution to the transposed equation

$$\int_{-a}^{a}\frac{[a_2 + \operatorname{sign}(s-x)\,a_1]^{\mu}\,g^*(s,a)\,ds}{|s-x|^{1-\mu}} = 1 \tag{2.14}$$

and

$$M(a) = \int_{-a}^{a} g(s,a)\,ds \tag{2.15}$$

Thus, if solutions to equations (2.13) and (2.14) are known, then according to (2.12) the determination of the contact pressure $p(x)$ reduces to quadratures.

3. *The pressure under a rigid die on a half-plane under conditions of steady creep when frictional forces are taken into account.* Consider the problem of a rigid die with a rectilinear base pressed on to a half-plane under conditions of steady creep, taking into account frictional forces.

Suppose that a rigid die is pressed on to a half-plane (Fig. 3). We shall assume that the die is in a state of limiting equilibrium, i.e. that a horizontal force $Q = kP$ acts on it in the direction of the x-axis. Then from (2.9) for this case we have

$$f_0(x) = 0, \qquad F(x,\gamma) = \gamma^{\mu} \tag{2.16}$$

and the integral equation (2.10) assumes the form

$$\int_{-a}^{a}\frac{[a_2 - \operatorname{sign}(s-x)\,a_1]^{\mu}\,p(s)\,ds}{|s-x|^{1-\mu}} = \gamma^{\mu} \tag{2.17}$$

On the basis of (2.13) and (2.17) we evidently have that

$$p(x) = \gamma^{\mu} g(x, a) \tag{2.18}$$

Substituting this expression for $p(x)$ in (2.11) and (2.18) we find

$$\gamma^{\mu} = P\left(\int_{-a}^{a} g(x, a)\, dx\right)^{-1}, \qquad p(x) = Pg(x, a)\left(\int_{-a}^{a} g(x, a)\, dx\right)^{-1} \tag{2.19}$$

We now determine the function $g(s, a)$, i.e. we solve the singular integral equation (2.13).

We shall try to find a solution to this equation in the form

$$g(s, a) = \frac{N}{\sqrt{(a^2 - s^2)^{\mu}}}\left(\frac{a+s}{a-s}\right)^{1/2\mu - \rho} \tag{2.20}$$

where N, ρ are unknown quantities and $0 < \rho < \mu$. Substituting this expression for $g(s, a)$ into equation (2.13), we obtain

$$\int_{-a}^{x} \frac{(a_2 + a_1)^{\mu} N}{(x-s)^{1-\mu}\sqrt{(a^2 - s^2)^{\mu}}}\left(\frac{a+s}{a-s}\right)^{1/2\mu-\rho} ds + \int_{x}^{a} \frac{(a_2 - a_1)^{\mu}}{(s-x)^{1-\mu}\sqrt{(a^2 - s^2)^{\mu}}}\left(\frac{a+s}{a-s}\right)^{1/2\mu-\rho} ds \tag{2.21}$$

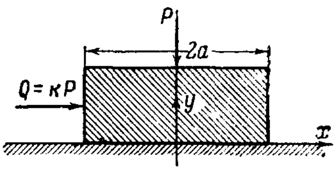

Fig. 3.

It can easily be shown that for the equality (2.21) to be satisfied it is necessary and sufficient for the constants N and ρ to have the following values*:

$$N = \frac{H}{\pi}, \quad \rho = \frac{1}{\pi} \sin^{-1} (a_1 + a_2)^{\mu} H \tag{2.22}$$

where

$$H = \frac{\sin \pi\mu}{\sqrt{(a_2 + a_1)^{2\mu} + 2(a_2^2 - a_1^2)^{\mu} \cos \pi\mu + (a_2 - a_1)^{2\mu}}} \tag{2.23}$$

Substituting the expression for $g(s, a)$ given by (2.20) into (2.19) and rearranging, we obtain the following formula for the pressure $p(x)$

* Note that the integral equation (2.14) can be solved by the method evolved in [3,4]. Here the parameters N and ρ are determined by a different method which was kindly indicated to the authors by I.D. Zaslavskii.

on the area of contact under the die:

$$p(x) = \frac{\Gamma(^1/_2(3-\mu))\,\Gamma(1-{}^1/_2\mu)\,\Gamma(\mu-\rho)\sin\pi(\mu-\rho)}{a^{1-\mu}\Gamma(1-\rho)} \times$$
$$\times \frac{P}{\pi\sqrt{(a^2-x^2)^{\mu}}}\left(\frac{a+x}{a-x}\right)^{^1/_2\mu-\rho} \qquad (2.24)$$

Here $\Gamma(s)$ is the gamma-function and ρ is a constant which can be found from formula (2.22).

It is not difficult to see that if friction is not taken into account, then $a_1 = 0$ and consequently $\rho = 1/2\ \mu$. We then find from (2.24) that

$$p(x) = \frac{\Gamma(^1/_2(3-\mu))\Gamma(^1/_2\mu)\sin(^1/_2\pi\mu)}{a^{1-\mu}\sqrt{\pi}}\ \frac{P}{\pi\sqrt{(a^2-x^2)^{\mu}}} \qquad (2.25)$$

which coincides with the solution obtained in [3] to the problem of a rigid die with a rectilinear base pressed on to a half-plane under conditions of steady creep and in the absence of friction.

TABLE 2.

μ	k	$x = {}^1/_4 a$	$x = {}^1/_2 a$	$x = {}^3/_4 a$
1	0	1.0360	1.1500	1.5100
	0.1	1.0360	1.1500	1.5100
0.70	0	1.2216	1.3209	1.5950
	0.1	1.2262	1.3325	1.6212
0.65	0	1.2501	1.3443	1.6024
	0.1	1.2582	1.3637	1.6431
0.30	0	1.466	1.638	2.144
	0.1	0.974	1.072	0.982
0.15	0	1.5406	1.7228	2.2557
	0.1	1.3112	1.4120	1.5859
0.20	0	1.5190	1.6986	2.2241
	0.1	1.1049	1.4075	1.1975
0.25	0	2.4978	1.6749	2.193
	0.1	1.0345	1.2437	1.081

As an example, consider the numerical determination of the pressure under a rigid die with a rectilinear base pressed onto a half-plane under conditions of nonlinear creep with friction taken into account.

Table 2 gives values of the pressure

$$\frac{P(x)}{\alpha}\quad\left(\alpha = \frac{P}{\alpha\pi}\right)$$

at the points $x = 1/4\ a$, $x = 1/2\ a$ and $x = 3/4\ a$ for various values of the creep exponent μ for $k = 0$ and $k = 0.1$.

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